Exercise 68

(a) By differentiating the double-angle formula

$$\cos 2x = \cos^2 x - \sin^2 x$$

obtain the double-angle formula for the sine function.

(b) By differentiating the addition formula

$$\sin(x + a) = \sin x \cos a + \cos x \sin a$$

obtain the addition formula for the cosine function.

Solution

Part (a)

Differentiate both sides of the given double-angle formula with respect to x.

$$\begin{aligned}
\frac{d}{dx}(\cos 2x) &= \frac{d}{dx}(\cos^2 x - \sin^2 x) \\
(-\sin 2x) \cdot \frac{d}{dx}(2x) &= \frac{d}{dx}(\cos^2 x) - \frac{d}{dx}(\sin^2 x) \\
(-\sin 2x) \cdot (2) &= \frac{d}{dx}(\cos x)^2 - \frac{d}{dx}(\sin x)^2 \\
-2\sin 2x &= 2(\cos x)^1 \cdot \frac{d}{dx}(\cos x) - 2(\sin x)^1 \cdot \frac{d}{dx}(\sin x) \\
&= 2\cos x \cdot (-\sin x) - 2\sin x \cdot (\cos x) \\
&= -2\cos x \sin x - 2\sin x \cos x \\
&= -4\sin x \cos x
\end{aligned}$$

Therefore, dividing both sides by -2,

$$\sin 2x = 2\sin x \cos x.$$

Part (b)

Differentiate both sides of the given angle addition formula with respect to x.

$$\frac{d}{dx}[\sin(x+a)] = \frac{d}{dx}(\sin x \cos a + \cos x \sin a)$$

$$\cos(x+a) \cdot \frac{d}{dx}(x+a) = \frac{d}{dx}(\sin x \cos a) + \frac{d}{dx}(\cos x \sin a)$$

$$\cos(x+a) \cdot (1) = (\cos a)\frac{d}{dx}(\sin x) + (\sin a)\frac{d}{dx}(\cos x)$$

$$\cos(x+a) = (\cos a)(\cos x) + (\sin a)(-\sin x)$$

Therefore,

$$\cos(x+a) = \cos a \cos x - \sin a \sin x.$$